CLASS 1

Tuesday, May 5, 2020 4:04 PM

Electric Potential.

* Electric potential : The work done to bring a unit +ve charge from ∞ to a given point is called EP at that point

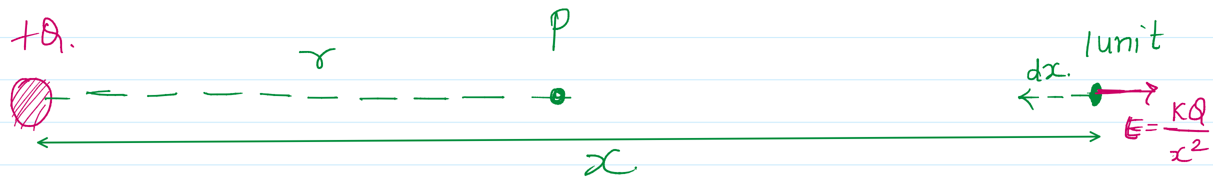
  * $V = \frac{\text{work}}{\text{charge}}$

  * unit J/C (or) volt (V)

  * Scalar quantity.

$$\boxed{\begin{aligned} W &= \vec{F}\cdot\vec{S} \\ &= FS\cos\theta \end{aligned}}$$

* Potential at a point due to point charge.



∴ small work done to move the unit +ve charge by a small dist dx

$$dw = E(-dx)$$

$$dw = -\frac{KQ}{x^2}(dx)$$

up on integrating from ∞ to r we get net work done.

$$\therefore \quad V = -\int_{\infty}^{r} \frac{KQ}{x^2}\,dx$$

$$= -KQ\int_{\infty}^{r} \frac{1}{x^2}\,dx$$

$$= -KQ\int_{\infty}^{r} x^{-2}\,dx$$

$$= -KQ\left.\frac{x^{-2+1}}{-2+1}\right|_{\infty}^{r}$$

$$= KQ\left.\frac{1}{x}\right|_{\infty}^{r}$$

0

$$= KQ\left[\frac{1}{r} - \cancel{\frac{1}{\infty}}^{\,0}\right]$$

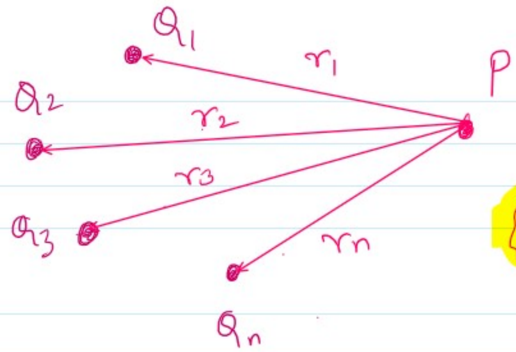
$$\boxed{V = \frac{KQ}{r}}$$

<u>NOTE</u>: ① potential at a point due to +ve charge is +ve & due to −ve charge it is −ve.

② If a charge "q" is kept at a point where the potential is V, then
PE of the charge q is $\boxed{Vq}$

* V is work per unit
∴ to bring a charge "q" from ∞ to a given point, the work done = Vq

Potential at a point due to system of charges.

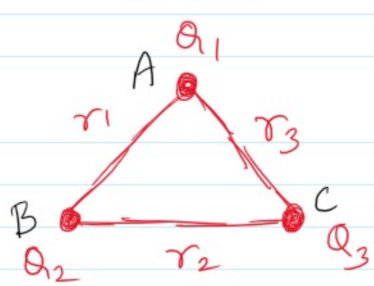


$$V = V_1 + V_2 + V_3 + \dots + V_n$$

$$\boxed{V = \frac{KQ_1}{r_1} + \frac{KQ_2}{r_2} + \dots + \frac{KQ_n}{r_n}}$$

Potential Energy of System of charges

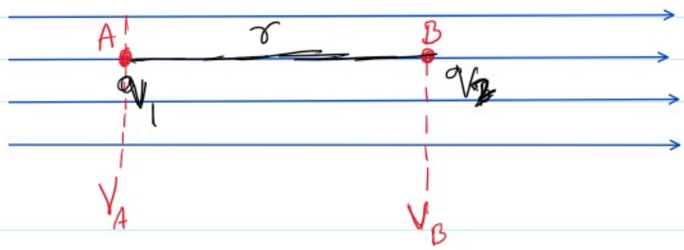
: The amount of work done inorder to assemble the charges is known as PES



$$PES = 0 + \frac{KQ_1}{r_1} \times Q_2 + \frac{KQ_1Q_3}{r_3} + \frac{KQ_2Q_3}{r_2}$$

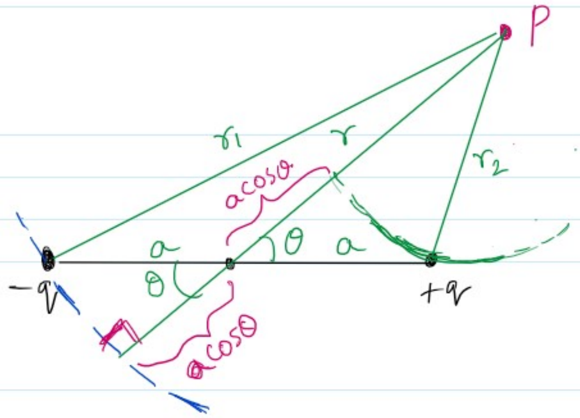
PE of System of 2 charges in External EF

$\longrightarrow E$



$$\boxed{PES = V_A q_1 + V_B q_2 + \frac{Kq_1 q_2}{r_1}}$$

Potential at a point due to Electric dipole



$$V_p = \frac{K(-q)}{r_1} + \frac{K(+q)}{r_2}$$

$$\boxed{V_p = Kq\left[\frac{1}{r_2} - \frac{1}{r_1}\right]}$$

$$\therefore \boxed{\begin{aligned} r_1 &= r + a\cos\theta \\ r_2 &= r - a\cos\theta \end{aligned}}$$

$$\therefore V_p = Kq\left[\frac{1}{r - a\cos\theta} - \frac{1}{r + a\cos\theta}\right]$$

$$V_p = Kq\left[\frac{r + a\cos\theta - (r - a\cos\theta)}{(r - a\cos\theta)(r + a\cos\theta)}\right]$$

$$V_p = Kq\left[\frac{2a\cos\theta}{r^2 - a^2\cos^2\theta}\right]$$

for a short dipole $a << r$ $\therefore r^2 - a^2\cos^2\theta \approx r^2$

$$V_p = \frac{Kq(2a\cos\theta)}{r^2} \rightarrow \boxed{V_p = \frac{PK\cos\theta}{r^2}}$$

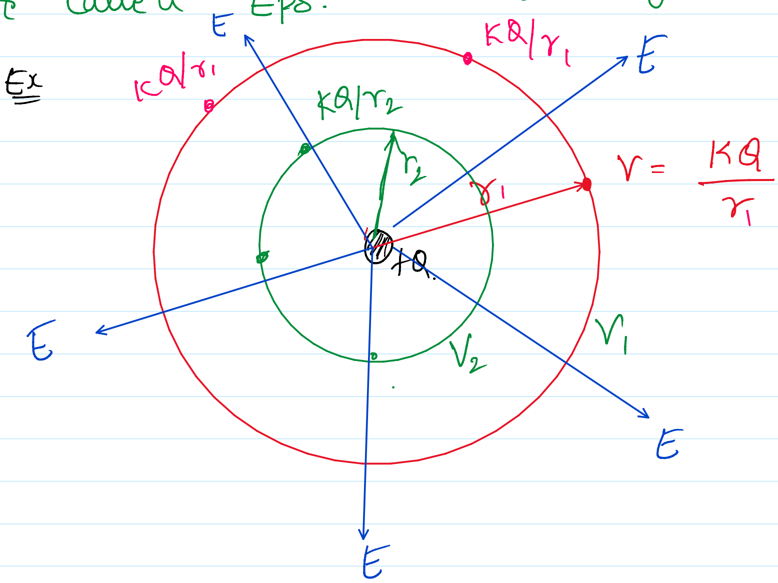
NOTE

* if $\theta < 90°$ then potential is +ve
* if $\theta > 90°$ then potential is −ve
* if $\theta = 90°$ then potential $= 0$

Equipotential surface-

The surface passing through the points having same potential is called EPS.

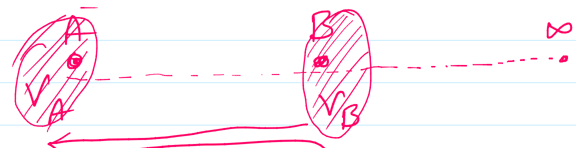
Ex



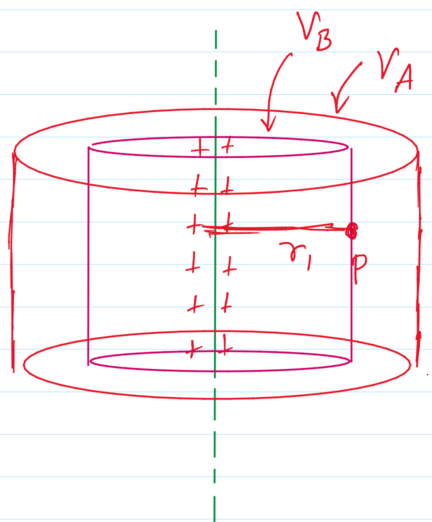
The shape of EPS is spherical due to point charge.

$V_2 > V_1$

Potential difference

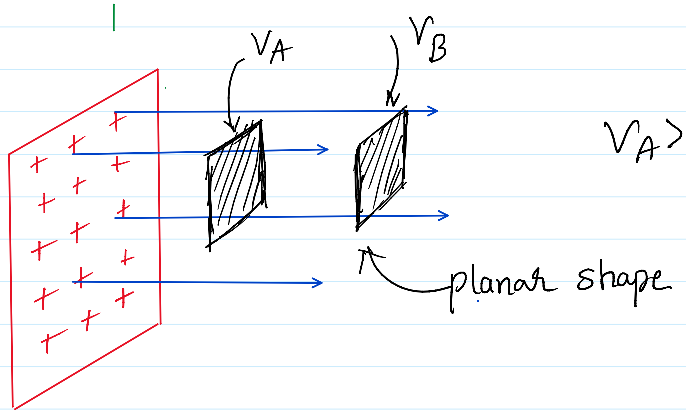


$V_A - V_B = pd$



The shape of EPS due to long wire carrying charge is cylindrical.

$V_B > V_A$



$V_A > V_B$

Properties of Equipotential Surface

* EF is always $\perp^{r}$ to EPS.

* Work done to move a charge on the surface of EPS $= 0$.

* Work done to move a charge on the surface of EPS $= 0$.

* Every surface of a charged conductor is an equipotential surface

Relationship b/w EF & p.d

$$\boxed{E = -\frac{dV}{dx}\hat{i}}$$

More general formula

$$\boxed{\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]}$$

$\frac{\partial V}{\partial x}$ Partial differentiation of V wrt x

Example

let $V = 3x^2y - 4zy^2 + 8xz^2$ be the potential in space. find the magnitude of EF at $(1,1,1)$.

$\partial V/\partial x$

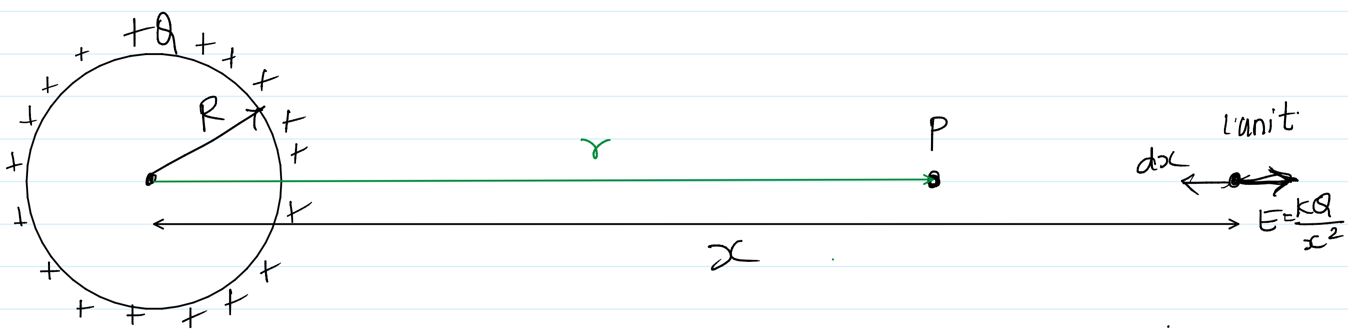
$$\vec{E} = -\left[(3y\,2x - 0 + 8z^2)\hat{i} + (3x^2 - 4z(2y) + 0)\hat{j} + (0 - 4y^2 + 16xz)\hat{k}\right]$$

$$\vec{E} = -\left[(14)\hat{i} + (-5)\hat{j} + (12)\hat{k}\right]$$

$$\boxed{\vec{E} = -14\hat{i} + 5\hat{j} - 12\hat{k}}$$

$$|\vec{E}| = \sqrt{(-14)^2 + (5)^2 + (-12)^2}$$

Potential at a point due to sphere carrying a charge



$$\boxed{dW = E(-dx)}$$

$$W_{net} = -\int^{r} E\,dx$$

∞

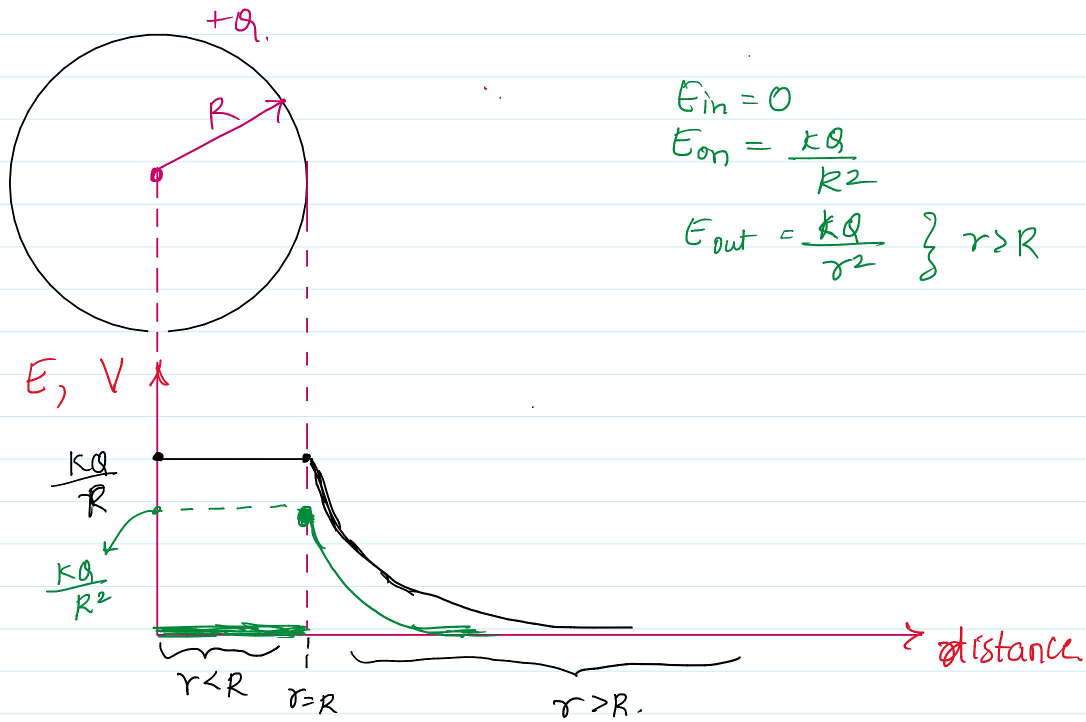
$$\boxed{V = \frac{KQ}{r}}$$

$\therefore$ $V_{outside} = \frac{KQ}{r}$

$V_{on\,the\,surf} = \frac{KQ}{R}$

$V_{inside} = V_{on\,the\,surface} = \frac{KQ}{R}$

Graph of EP vs distance for a spherical body carrying charge.

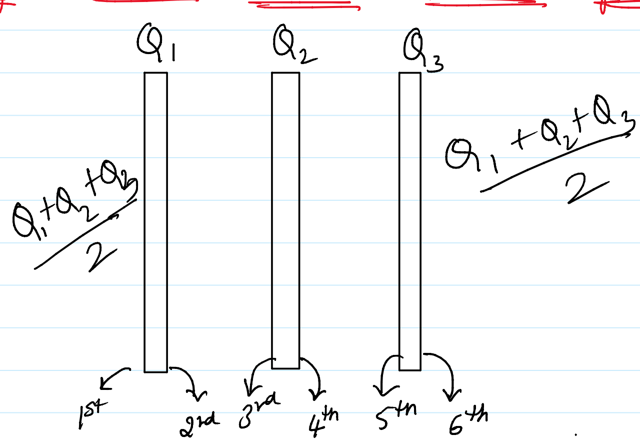


$E_{in} = 0$

$E_{on} = \frac{KQ}{R^2}$

$E_{out} = \frac{KQ}{r^2}$ } $r > R$

Charge distribution on the charges plates



$1^{st} = \frac{Q_1 + Q_2 + Q_3}{2}$

2^{nd} surface $= Q_1 - \left[\frac{Q_1 + Q_2 + Q_3}{2}\right]$

$= \frac{Q_1 - Q_2 - Q_3}{2}$

3^{rd} surface $= -$ charge on 2^{nd}

$= \frac{Q_2 + Q_3 - Q_1}{2}$

$$4^{th}\ \text{Surface} = Q_2 - \left[\frac{Q_2+Q_3-Q_1}{2}\right]$$

$$= \frac{Q_2 - Q_3 + Q_1}{2}$$

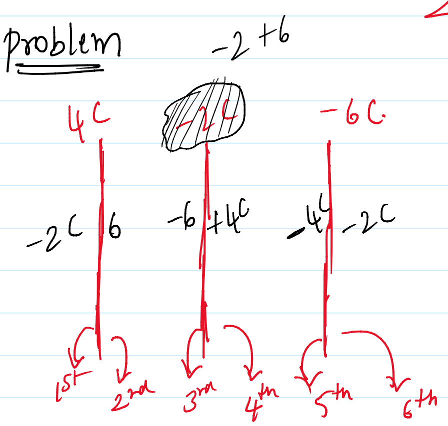
$$\therefore\ 5^{th}\ \text{surface} = -\ \text{Charge on}\ 4^{th}$$

$$= \frac{Q_3 - Q_2 - Q_1}{2}$$

$$6^{th}\ \text{surface} = Q_3 - \left[\frac{Q_3 - Q_2 - Q_1}{2}\right]$$

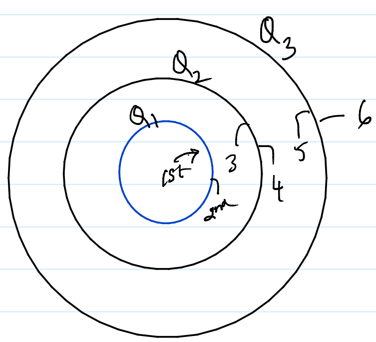
$$= \frac{Q_1 + Q_2 + Q_3}{2}$$

__Sample problem__



1^{st} & 6^{th} $= \underline{\underline{-2C}}$

Charge distribution in case of sphere



1^{st} : 0

2^{nd} : Q_1

3^{rd} : $-Q_1$

4^{th} : $Q_2 - (-Q_1)$

$Q_2 + Q_1$

5^{th} : $-(Q_2 + Q_1)$

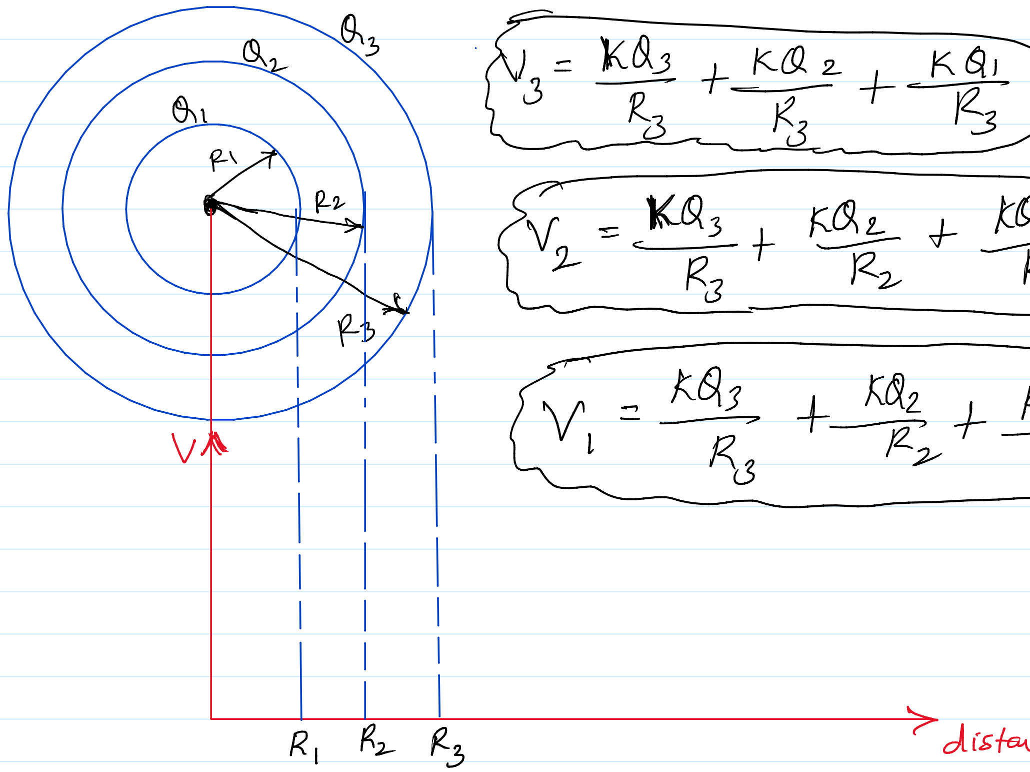
6^{th} : $Q_3 + [-(Q_1 + Q_2)]$

$\underline{\underline{Q_1 + Q_2 + Q_3}}$

Concept of Earthing (or) grounding (⏚)

Whenever a body is grounded, then the potential of that

Whenever a body is grounded, then the potential of that body $= 0$.



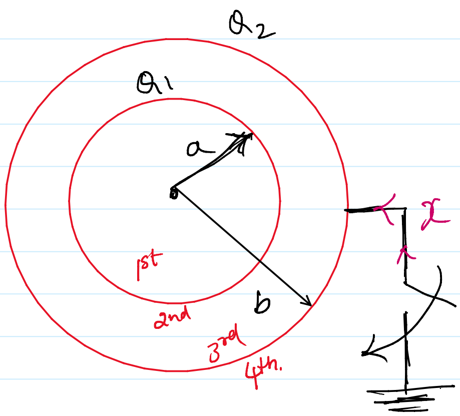
$$V_3 = \frac{KQ_3}{R_3} + \frac{KQ_2}{R_3} + \frac{KQ_1}{R_3}$$

$$V_2 = \frac{KQ_3}{R_3} + \frac{KQ_2}{R_2} + \frac{KQ_1}{R_2}$$

$$V_1 = \frac{KQ_3}{R_3} + \frac{KQ_2}{R_2} + \frac{KQ_1}{R_1}$$

Sample problem on grounding

(1)



* Find the charge that flows through the switch after closing the switch.
* Also comment what flows from where to where.
* write the charge on each surface of the sphere

Solution.

let x be the amount of charge that flows from the ground to the 2nd sphere to make the potential of 2nd sphere zero.

$$\cancel{V_2}^{\,0} = \frac{K(Q_2+x)}{b} + \frac{KQ_1}{b}$$

$$\therefore \quad Q_2 + x = -Q_1$$

$$x = -Q_1 - Q_2$$

$$\therefore \boxed{x = -(Q_1 + Q_2)}$$

Charge on 1st surface $= 0$

2nd surface $= Q_1$

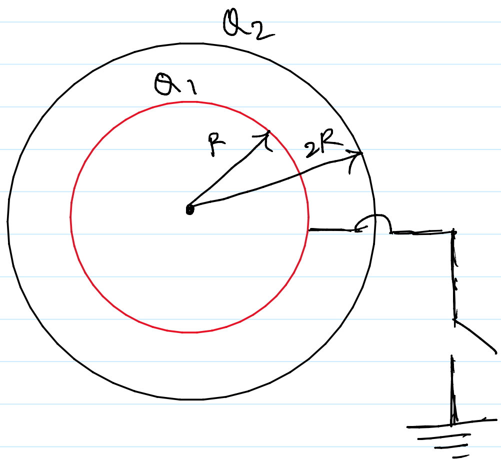
3rd surface $= -Q_1$

4th surf $= (Q_2 + x) - (-Q_1)$

$= Q_2 - (Q_1 + Q_2) + Q_1$

$= 0$

(2)



* find charge the flows through the switch
* what flows from where to where
* Charge on each surface of the sphere.